Modified Rathbun Analysis

Start with Rathbun's 4 basic statistical properties.

Bayes Theorem:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} \qquad [1]$$

Probability for intersecting sets:

$$P(A\&B) = P(A \mid B)\ P(B) \qquad [2]$$

Probability when A and B are independent :

$$P(A\&B) = P(A)P(B) \qquad [3]$$

Decomposing A into its intersection with B and $(notB) = B^{-1}$

$$P(A) = P(A\&B) + P(A\&B^{-1}) \qquad [4]$$

Let H be a hypothesis, and $E_i \;\; i = 1, 2, \dots, n$ a set of n pieces of evidence that relate to H.

Follow Rathbun's steps within his Eq (1).

Using [1]

$$P(H \mid E_1\&E_2\dots\&E_n) = \frac{P(E_1\&E_2\dots\&E_n \mid H)\ P(H)}{P(E_1\&E_2\dots\&E_n)} \qquad [5]$$

Using [4] in the denominator

$$P(H \mid E_1\&E_2\dots\&E_n) = \frac{P(E_1\&E_2\dots\&E_n \mid H)\ P(H)}{P(E_1\&E_2\dots\&E_n\&H) + P(E_1\&E_2\dots\&E_n\&H^{-1})} \qquad [6]$$

Using [2] for each of the 2 terms in the denominator

$$P(H \mid E_1\&E_2\dots\&E_n) = \frac{P(E_1\&E_2\dots\&E_n \mid H)\ P(H)}{P(E_1\&E_2\dots\&E_n \mid H)P(H) + P(E_1\&E_2\dots\&E_n \mid H^{-1})P(H^{-1})} \qquad [7]$$

Using [3] to distribute the n independent pieces of evidence in both numerator and denominator

$$P(H \mid E_1\&E_2\dots\&E_n) = \frac{R}{R+S} \qquad [8]$$

$$R = [P(E_1 \mid H)P(E_2 \mid H)\dots P(E_n \mid H)]\ P(H) \qquad [8a]$$

$$S = [P(E_1 \mid H^{-1})P(E_2 \mid H^{-1})\dots P(E_n \mid H^{-1})]\ P(H^{-1}) \qquad [8b]$$

Rewriting [8] using Π product notation

$$P(H \mid E_1 \& E_2 \ldots \& E_n) = \frac{P(H) \prod_{i=1}^{n} P(E_i \mid H)}{P(H) \prod_{i=1}^{n} P(E_i \mid H) + P(H^{-1}) \prod_{i=1}^{n} P(E_i \mid H^{-1})} \qquad [9]$$

Equation [9] above is the final identity in Rathbun's Eq (1). Now depart from the original analysis by interpretting

$$p_i = P(E_i \mid H) \qquad [10]$$

as the estimated probability that evidence E_i supports that the hypothesis H is true and

$$1 - p_i = P(E_i \mid H^{-1}) \qquad [11]$$

as the estimated probability that evidence E_i supports that the hypothesis H is false.

$P(H)$ is the prior probability of H being true before any of the evidence E_i is considered. $P(H^{-1}) = 1 - P(H)$ is the prior probability of H being false before any of the evidence E_i is considered.

Substituting [10] and [11] into [9]

$$P(H \mid E_1 \& E_2 \ldots \& E_n) = \frac{P(H) \prod_{i=1}^{n} p_i}{P(H) \prod_{i=1}^{n} p_i + P(H^{-1}) \prod_{i=1}^{n} (1 - p_i)} \qquad [12]$$

If there is no prior knowledge of H, then $P(H) = P(H^{-1}) = 0.5$, and these factors then cancel from the numerator and denominator of [12] to give

$$P(H \mid E_1 \& E_2 \ldots \& E_n) = \frac{\prod_{i=1}^{n} p_i}{\prod_{i=1}^{n} p_i + \prod_{i=1}^{n} (1 - p_i)} \qquad [13]$$